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BUCKLING ANALYSIS OF SHUTTLE DISPOSABLE LIQUID HYDROGEN TANK WITH FLOATING RINGS

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Robert Winter
Jacques Crouzet-Pascal

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by

R. Winter

and

J. Crouzet-Pascal

Materials and Structural Mechanics

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Approved by: Charles E. Mack, Jr.

Charles E. Mack, Jr.
Director of Research

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A scheme for treating so-called floating rings is recommended for use in the buckling analysis of stiffened cylindrical shells. Critical stresses are calculated and compared to those for integral rings, for a design representative of the unpressurized Space Shuttle liquid hydrogen (LH_2) tank. The ring rigidity required to prevent general instability is found to be much less than that required by the Shanley criterion, with a correspondingly significant weight saving. There is very little difference in total shell weight between floating and internal integral rings for equal strength designs.

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LIST OF SYMBOLS

A_r, A_s	area of ring and stringer cross sections (in.^{2})
d_r, d_s	spacing of rings and stringers (in.)
E	Young's modulus (psi)
I	bending moment of inertia of unstiffened shell wall, $t^3/12$ (in.^{3})
I_r, I_s	bending moment of inertia of rings and stringers about their centroids (in.^{4})
$\bar{I}_r$, $\bar{I}_s$	bending moment of inertia of rings and stringers about shell midsurface (in.^{4})
J_r, J_s	torsional moment of inertia of rings and stringers, (in.^{4})
L	length of shell (in.)
m	number of longitudinal halfwaves in buckle shape
n	number of circumferential waves in buckle shape
R	shell radius (in.)
t	shell skin thickness (in.)
t_x	average wall thickness in longitudinal direction, $t + A_s/d_s$ (in.)
$\bar{z}_r$, $\bar{z}_s$	eccentricity of rings and stringers, distance from centroids to shell midsurface (in.)
μ	Poisson's ratio
λ_x, λ_y	wavelengths of buckle shape, longitudinal and circumferential (in.)
σ_x	critical stress (psi)

INTRODUCTION

Compressive load-carrying cylindrical shells for lightweight aerospace structures are usually made with circumferential and longitudinal stiffening elements (rings and stringers) that are continuously attached to the thin shell wall, whether by closely-spaced rivets or by integral construction achieved by machining from thicker plates (Fig. 1). A less costly method has been proposed for the Shuttle Orbiter disposable LH_2 tank whereby the rings would be attached by pins to the free edge of each stringer (Fig. 2), the so-called "floating ring" design.

The purpose of this investigation was to determine the effect of using floating rings on the buckling strength and weight of a particular type of stiffened cylindrical shell that is being considered for the LH_2 tank. No attempt was made to design an actual structure; instead the effort was confined to finding a technique for analyzing the floating ring construction, and to estimate the magnitude of the important trends.

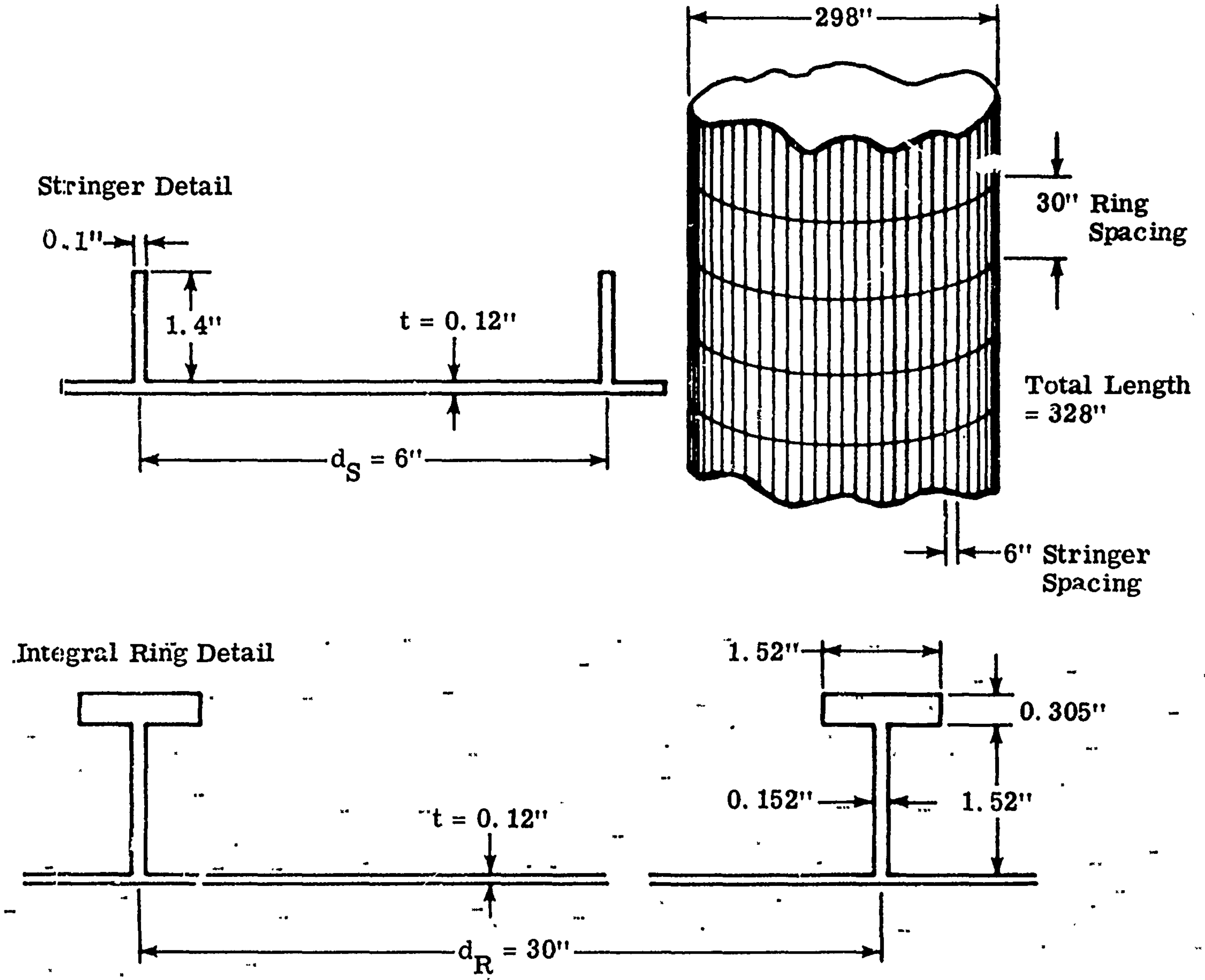

Fig. 1 Baseline Configuration.

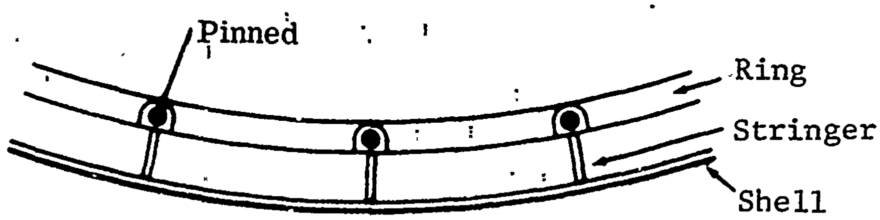

Fig. 2 Floating Ring Configuration.

DISCUSSION

Method of Calculation

Calculations of critical loads for general and panel instability modes were made for the stiffened cylinder described in Fig. 1 and Table 1, with integral rings and with floating rings,

Table 1

STIFFENER PARAMETERS FOR BASELINE DESIGNS

	Integral Rings	Floating Rings	Stringers
$\bar{I}_r$ (in.^{4})	2.83	2.83	
I_r (in.^{4})	1.46	2.83	
J_r (in.^{4})	0.0162	0	
A_r (in.^{2})	0.698	0*	
$\bar{z}_r$ (in.)	-1.40**	0	
d_r (in.)	30	30	
$\bar{I}_r/d_r I$	655	655	
$\bar{I}_s$ (in.^{4})			0.104
I_s (in.^{4})			0.0229
J_s (in.^{4})			4.67×10^{-4}
A_s (in.^{2})			0.14
$\bar{z}_s$ (in.)			-0.76**
d_s (in.)			6
$\bar{I}_s/d_s I$			120

*0.971 for Weight Calculations

**Negative for inside stiffeners

using Eq. (17) of NASA TND-2960 (Ref. 1). That report uses a method that averages (smears) the purely one dimensional stiffene properties over the shell wall. The stiffener eccentricity is considered, with different results often predicted for inside and outside stiffeners. The cylinder with floating rings was analyzed by setting equal to zero the ring torsional rigidity, cross-sectional area, and eccentricity, as discussed in Appendix A.

The usual procedure in designing for stability of a ring and stringer stiffened cylinder is to prevent both panel instability (between rings) and local skin buckling (between rings and stringers) by adjustment of stringer properties and ring spacing, compatible with the requirement of low weight. In this procedure, the rings are assumed to be rigid enough to provide simple support conditions to the panels. The rings are then designed to provide at least this necessary rigidity, with the final ring rigidity set larger than the required value so that the general instability strength (when rings also buckle) is somewhat higher than the panel strength. This is done to compensate for manufacturing imperfections or local stress variations that might reduce the general instability strength, even though the shell weight is increased.

A common practice in the determination of the minimum ring rigidity is to use a semiempirical method, established by Shanley (Ref. 2) and modified by Gerard (Ref. 3), in which only the ring bending rigidity is considered. This criterion can be expressed as

$$E_r \bar{I}_r d_r / 4\pi R^4 N_x > 6.85 \times 10^{-5}$$

or, in terms of the ring rigidity parameter, as

$$\bar{I}_r / d_r I > 8.6 \times 10^{-4} \, \sigma_x t_x R^4 / E_r I d_r^2 \quad .$$

A more rational approach would be to vary the ring properties, while holding the other shell characteristics constant, and to calculate the critical ring rigidities for which the weakest buckle mode changes from that for panel instability to that associated with general instability. This approach was taken by Block (Ref. 4), who showed that the Shanley ring criterion seemed to be unrelated to the actual requirements (at least for the cases he examined). He found that the critical ring rigidity was much less than Shanley's for shells that buckle into a large number of circumferential waves, and much larger than Shanley's for a small number of circumferential waves. Accordingly, we adopted this approach with regard to the present computational effort.

All of the numerical results are summarized in Fig. 3, in which the critical loads for general instability with integral rings and floating rings, and for panel instability, are plotted as functions of the ring bending stiffness parameter, $\bar{I}_r/d_r I$. The ring size scale factor is also shown; using the baseline ring size (Table 1) as full scale. Note that the ring stiffness parameter indicates the ring bending inertia referenced to the unstiffened shell midsurface, defined as

$$\bar{I}_r = I_r + \bar{z}_r^2 A_r \quad .$$

In the case of the floating ring, the eccentricity is ineffective (as discussed in Appendix A), so that $\bar{z}_r = 0$, and $\bar{I}_r = I_r$, the inertia about the ring centroid. Thus, in designing a ring by the Shanley method to prevent general instability, which requires a minimum $\bar{I}_r$, the floating rings will be heavier than the integral rings.

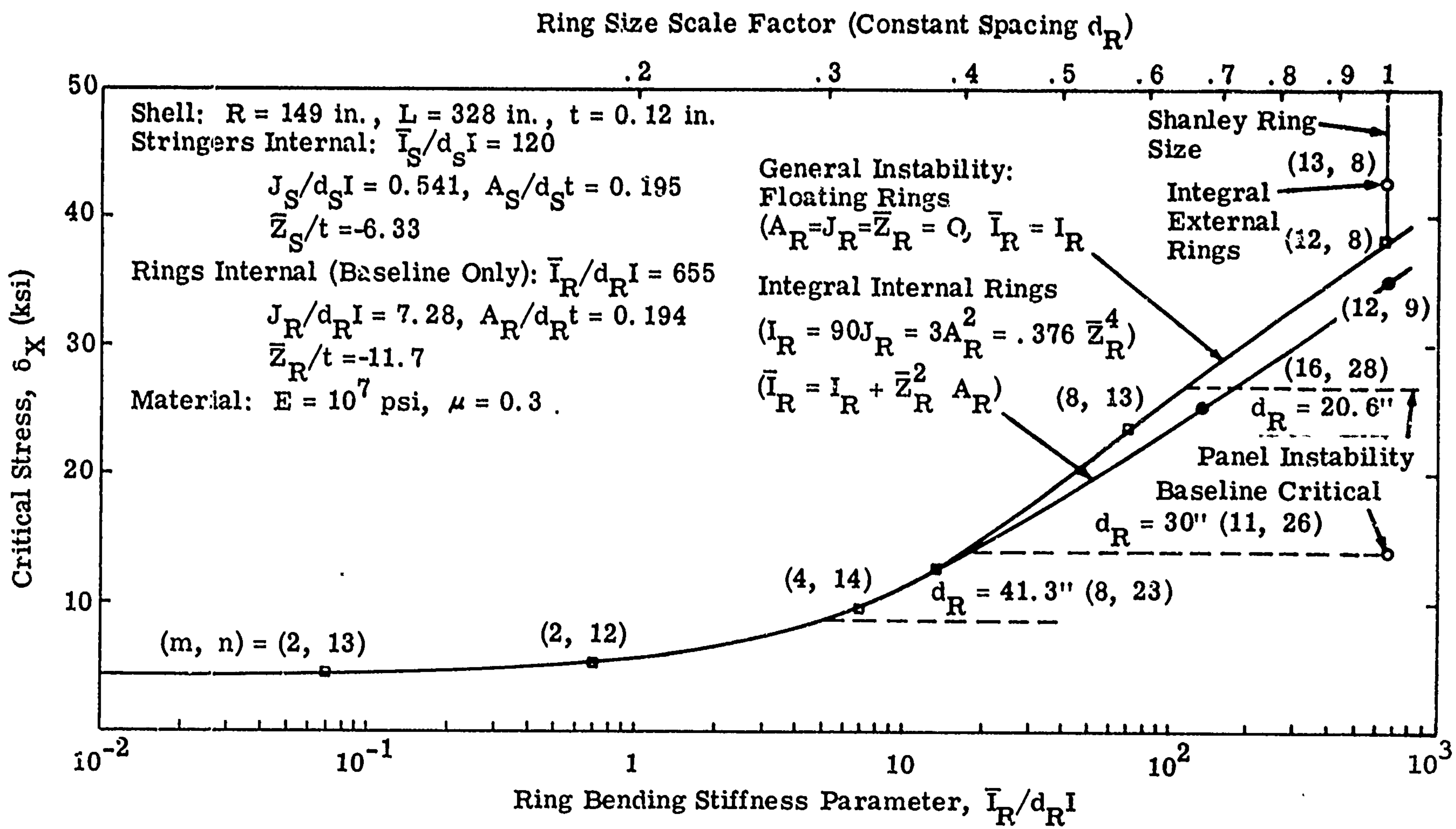

Fig. 3 Effect of Rings on Buckling of Stiffened Cylinder Under Uniform Axial Compression Using NASA TND-2960. EQ (17)

The general instability curve for floating rings was generated by varying the parameter, $\bar{I}_r/d_r I$. The curve for integral rings was generated by keeping the ring cross section shape constant, but reducing its size. Consequently, certain relations between the various ring parameters are maintained. These relations are:

$$I_r = 90\ J_r = 3\ A_r^2 = 0.376\ \bar{z}_r^4 \quad .$$

The general instability curves apply to any combination of ring inertia and spacing.

Since the total inertia, $\bar{I}_r$, of the ring as contributed to the shell was used in Fig. 3, it would be expected that the two curves for integral and floating rings would coincide; that they do not is attributed to the fact that for this case, inside (integral) rings produce a weaker shell than noneccentric (floating) rings. A calculation was made for outside integral rings that showed the greatest buckling strength. This point is plotted in Fig. 3, and shows the sensitivity to inside/outside eccentricity of this particular cylinder, as predicted by the theory of Ref. 1.

The theoretical panel instability results were calculated by assuming that the rings provided classical simple supports to a stringer-stiffened cylinder of length equal to the ring spacing. These values are shown for several ring spacings, including the baseline 30-inch spacing. The classical simple support condition assumes that there is no resistance to axial buckle deformation at the panel ends. In actuality, the integrally attached rings will resist this deformation due to their bending rigidity about an axis perpendicular to the shell surface. The floating rings can be assumed to be completely flexible in this mode. There is, however, an unavoidable resistance to this axial deformation provided

by the in-plane shear rigidity of the shell wall on the other side of the ring in the adjacent panel. The effect of this resistance is to increase the panel buckling strength above the value for classical simple supports, so that use of such a classical theory will be somewhat conservative. This axially elastic type of boundary condition on panel instability was incorporated by Block (Ref. 4) into his discrete-ring analysis, and he shows panel buckling loads that are larger than those given by the classical theory, but approach the classical values as the ring spacing decreases.

Numerical Results

For the case at hand, the baseline rings shown in Fig. 1 and Table 1 are dictated by the Shanley criterion, which indicates a ring inertia of $\bar{I}_r > 2.83$ in.^{4} ($\bar{I}_r/d_r I > 655$). Table 2 shows that the floating rings would be about 40 percent heavier than

Table 2

RELATIVE THICKNESSES (WEIGHTS) FOR d_r = 30 INCHES

	Ring Design				
	Baseline		Equal Strength	Light Rings of Equal Strength	
	Integral	Floating	Floating	Integral	Floating
$\bar{I}_r/d_r I$	655	655	420	65	50
Gen. Instab. stress (ksi)	35.1	38.2	35.1	21.0	21.0
$t + A_s/d_s$ (in.)	0.143	0.143	0.143	0.143	0.143
$A_r/d_r t$ (in.)	0.023	0.032	0.026	0.0068	0.0090
Avg. Wall Thick., $\bar{t}$ (in.)	0.166	0.175	0.169	0.150	0.152

the integral rings, for this baseline design, resulting in a greater total cylinder weight by about 6 percent. However, Fig. 3 shows that these floating rings produce a larger general instability stress than do the internal integral rings, because of the inside/outside effect. If the floating rings are designed to produce the same critical stress (35.1 ksi) for general instability as the integral Shanley rings, then they will be only 13 percent heavier than the integral rings, with a total shell weight only 2 percent greater. So, on the basis of weight required to provide equal strength, the floating and integral ring designs are about the same for the baseline case.

Figure 3 shows that the baseline ring design produces a general instability stress that is about 2.5 times the panel buckling stress of 14 ksi, with a 30-inch ring spacing. Reducing the general instability strength to only 1.5 times the panel strength should still prevent general instability, while reducing the required $\bar{I}_r$ of the floating rings to about 0.22 in.^{4} ($\bar{I}_r/d_r I = 50$). The corresponding floating ring weight (Table 2) is reduced to about 27 percent, and the ring dimensions are reduced to about 52 percent (half scale), of those for the baseline floating rings. The total shell weight would then be about 13 percent below that of the baseline floating ring design.

The curves of Fig. 3 show the theoretical predictions for the buckling behavior. However, design stresses might be expected to be lower, since thin unstiffened cylindrical shells under axial compression load produce actual buckling loads that can be far below the theoretical predictions.

There has been some small amount of test data (Refs. 5 and 6) that show that certain stiffened cylinders do not suffer this great reduction in buckling strength when the cylinders are either heavily ring stiffened or heavily longitudinally stiffened. These data show that only when the two stiffening systems are nearly in balance as measured, for example, by the effective radii of gyration in the two directions, are the test data as low as for the unstiffened isotropic shells. The better test performance with highly unbalanced stiffening systems has been noted, but often disregarded, perhaps because these tests were performed on small models with stiffeners not representative of actual construction practice (they were shallow, wide stiffeners of very small eccentricity).

Design factors are available, defined as the ratio of design critical load to theoretical critical load, which are based primarily on the poor performance of unstiffened isotropic cylinders, with some allowance for the orthotropy provided by the stiffening system (e.g., Refs. 7 and 8). For the cases exam[illegible]e, these design factors are in the range 0.25-0.35. While the load-carrying capacity is severely reduced it was found that the critical value of the ring inertia parameter to prevent general instability is not greatly changed.

SUMMARY AND CONCLUSIONS

An examination was made of certain design aspects of a stiffened cylindrical tank structure (such as the Shuttle Orbiter disposable LH_2 tank) for resisting buckling under axial loads. This investigation was undertaken to establish a method of analyzing the "floating" ring configuration, and to compare the floating ring and the integral ring configuration on the basis of buckling strength and weight.

An actual design analysis was not attempted. Instead, a representative (baseline) design was considered, under a uniform axial compression load, to illustrate the quantitative effects.

The floating ring configuration was analyzed by assuming that the rings contribute only their bending stiffness to the shell, and act as if their centroid coincides with the shell midsurface. This method, derived from physical considerations, is expected to be accurate for the type of shell design used.

Several available analysis methods were evaluated, including discrete rings versus smeared rings. It was found that, for the type of shells considered, the smeared ring theories are adequate for buckling analysis. Among those examined, the buckling formula of Ref. 1 [see Eq. (17)], currently in use at Grumman, is most realistic. This formula (coded in an existing Grumman FORTRAN program for the IBM 1130 computer) was used for the quantitative estimates.

As an illustrative example, the stiffened cylinder shown in Figs. 1 and 2 was examined. The results, given in Fig. 3, show the critical stress levels for general instability of the integral ring and floating ring designs and for panel instability, as

functions of the ring bending rigidity parameter or, alternatively, the ring size scale factor. The ring rigidity is varied from the baseline value obtained by the Shanley criterion (Ref. 2), down to zero to determine the minimum ring size to prevent general instability. Average shell wall thicknesses (a measure of relative weight) are shown in Table 2.

The major conclusions for the given tank structure are:

- The use of floating rings will not significantly change the critical buckling behavior.
- The baseline ring stiffness (from the Shanley criterion) is more than adequate to provide simple supports for panel buckling, so that significant weight savings (about 13 percent of total weight) may be achieved by reducing the floating ring dimensions by about 50 percent, without reduction of fundamental buckling strength.
- The total shell weight differs by only 2 percent between the floating ring and the internal integral ring designs, when equal strength is required.
- For the type of shell examined, a discrete ring theory is not required, and among the smeared ring, orthotropic, linear theories examined, that of Ref. 1 [Eq. (17)], seems to be the most realistic in treating the stiffeners

Design stress levels of 25 to 30 percent of the theoretical levels were calculated using the reduction factors recommended in Ref. 3. These low reduction factors are the result of assuming that the stiffened cylinder would behave as poorly, compared to the theoretical predictions, as an equivalent isotropic cylinder.

Some limited test data seem to indicate that this assumption may be overly conservative for designs examined here. Although all the stress levels were greatly reduced, the major conclusions (stated above) did not change when these reduction factors were applied.

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APPENDIX A

FLOATING RING ANALYSIS

The method of analyzing the floating rings was derived solely from physical arguments, as follows: The rings are assumed to be pinned at their centroid to the free edge of each stringer (Fig. 2). The pinned joint is assumed to transmit no moments, but only forces acting in the plane of the ring. Thus, the rings would not contribute their torsional rigidity to the shell. The stringers are assumed to be very flexible against circumferential forces at the pins, but to be very rigid for radial forces. This is analogous to making the stringers into rigid links pinned at both ends. Consequently, the rings will not resist any localized circumferential forces (arising from purely bending deformation of the shell). The rings will still resist extensional shell deformations as in an over-all radial motion of the shell ("breathing" mode). However, these extensional deformations are not significant for buckle modes that produce more than four waves around the circumference $(n > 4)$, which is common for most shell designs. Therefore, the rings would not contribute their extensional rigidity to the shell.

All that remains to be considered is the circumferential bending interaction. If the stringer spacing, d_s, is sufficiently less than the circumferential half-wavelength, $\lambda_y/2$, of the buckle deformation (as shown to be the case below for the present baseline design), then the ring centroidal axis and the shell will have the same radial deformation. Thus, the ring would contribute to the shell its full bending rigidity taken about its centroid (the ring would act effectively as if attached to the shell mid-surface).

Therefore, the floating ring is entered into the buckling analysis by neglecting its torsional rigidity, J_r, cross sectional area, A_r, and eccentricity, $\bar{z}_r$, while retaining fully its bending rigidity, I_r. This method should be reasonably accurate for $d_s < \lambda_y/2 = \pi R/n$, although the pin joint might transmit some torsional moment, and the rigidity of the stringers will produce some local circumferential loads, permitting the rings to contribute some finite effective J_r and A_r.

If the circumferential buckle wavelength is very large, extension of the ring will occur because of over-all radial deformation of the shell, and the ring cross sectional area cannot be neglected. The condition for use of the above floating ring assumptions to produce reliable results can be stated as follows in terms of the circumferential wave number n:

$$\pi R/d_s > n > 4 \quad .$$

Satisfaction of this condition can be checked in the analysis. For the example of Fig. 1, $\pi R/d_s = 78$ and all the values of n in Fig. 3 fall safely within these limits.

APPENDIX B

COMPARISON OF THEORIES

Discrete Versus Smeared Rings

The traditional method of analyzing the buckling of ring and stringer stiffened shells is to calculate the effective rigidity of the shell wall by adding the stiffener rigidity, averaged over the stiffener spacing, to the skin rigidity. The shell is then treated as an equivalent shell having continuous rigidity. Thus, the local effects of the stiffener-shell interaction are neglected. This method results in the convenient direct representation of the buckle load as a function of the buckle wave numbers. However, the fundamental neglect of local effects is valid only when the buckle half-wavelength is much larger than the stiffener spacing, since there would be many stiffeners deformed by a single buckle inward or outward.

In the case of many buckles between stiffeners, the stiffeners have a reduced effect on the buckle deformation and, therefore, play a much reduced role in stabilizing the shell, while the assumption of smeared stiffeners still considers the stiffener fully effective in an average sense. Consequently, the smeared stiffener theories will overpredict the strength of the shell in such cases, and a discrete-stiffener theory is required.

In typical aerospace-type stiffened cylinders, the stringers are almost always closely spaced but the rings are relatively far apart. Consequently, smeared stringer theory is sufficient to analyze panel buckling between rings, but the general instability calculations may require a discrete ring theory. The discrete ring theories are usually more complex than smeared ring theories

and require numerical methods for their solution. Several researchers (Refs. 4, 9, 10, and 11) have used such theories and some have compared results with the smeared orthotropic cylinder theories. In Refs. 4, 9, and 10, only small differences were reported for cases where the orthotropic theory predicts a half-wavelength as small as 1.2 times the ring spacing. No comparisons were found, in the short time spent on this investigation, for cases where the half-wavelength was equal to or less than the ring spacing.

In the baseline case examined here, the above situation occurs whereby the use of a smeared ring theory is questionable. However, the panel buckling modes become critical at a much lower load than do the general instability predictions. Therefore, a more accurate general instability theory is not needed for the baseline case. For rings less rigid than the baseline case, this wavelength problem is not encountered, since the longitudinal half-wavelength is greater then the ring spacing ($m < 11$). Consequently, a discrete-ring theory is not required for the present study.

Stiffener Transverse Rigidity

It is common practice, in the buckling analysis of shells, to replace a stiffened shell having geometric orthotropy by an equivalent thin shell having only material orthotropy. In this case, the equivalent coefficients of material orthotropy are calculated from the geometry and material of the shell plus the smeared stiffeners. Such a theory reduces the stiffener elements to an equivalent shell surface that has finite resistance to in-plane loads and moments imposed perpendicularly to the original stiffener direction. In the case of the most common constructions used today,

with the stiffener web width being a very small fraction of the stiffener spacing, the stiffeners can be considered purely one dimensional elements, contributing little to the shell bending and membrane rigidities in the direction perpendicular to their axis.

For noneccentric stiffeners, the equivalent orthotropic shell theories overestimate the rigidities of the shell wall, by adding terms from the transverse in-plane rigidities of the stiffeners such as

$$\mu\left[\frac{E_s I_s}{d_s}\right]\left[\frac{n}{R}\right]^2 \quad , \quad \mu\left[\frac{E_r I_r}{d_r}\right]\left[\frac{n}{R}\right]^2 \quad ,$$

to the buckle load n_x. Such terms can be significant. For the case of the panel buckling of the shell given in Fig. 1, these terms increase the buckle load with noneccentric stiffeners by about 33 percent.

For the case of eccentric stiffeners, the results are less clear. There would be a coupling between membrane forces and curvatures, and between moments and extensions. For example, a transverse membrane tensile force in the skin would cause a Poisson's contraction in the skin along the stiffener axis. Narrow webbed stiffeners would not experience the transverse stress, and so would resist the Poisson's contraction, thereby causing a tendency for the shell to deflect nonuniformly between stiffeners. This might tend to lower the effective rigidity of the shell wall, and at the same time it would cause a radial pre-buckle deformation that might tend to lower the actual buckle load. Therefore, a more exact analysis of an eccentrically stiffened shell would require a nonlinear treatment, analogous to that of

the beam column buckling behavior. Since no such buckling analyses were found during this investigation, the practical importance of this coupling effect was not determined.

Some test results on stiffened cylinders were reported in Refs. 5 and 6 that compared well with corresponding results from the equivalent orthotropic shell theory. However, the specimens were made specifically to suit that theory, with wide, shallow, almost noneccentric stiffeners having an effective resistance to transverse in-plane loads, and were not representative of typical aerospace structures.

The theoretical treatment of Block et al. (Ref. 1) properly does not add transverse in-plane rigidity to the stiffeners, but neglects the coupling forces transverse to, and curvatures along, the stiffener axis. It also neglects similar coupling between moments and extensions, and the associated nonlinear effects. Theoretically, their Eq. (17) (currently in use at Grumman as a design and analysis tool, programmed for the IBM 1130 computer) is strictly accurate only for the case of noneccentric stiffeners. Nevertheless, among the available linear smeared stiffener theories, it is the most realistic for analyzing the buckling behavior of ring- and stringer-stiffened cylinders of practical construction.